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CAPM

$$\begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \quad \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$$E[R_i] = R_f + \beta_{im}(E[R_m] - R_f) \quad (1)$$

$$\beta_{im} = \frac{\text{Cov}[R_i, R_m]}{\text{Var}[R_m]} \quad (2)$$

R_f R_m

i Z_i
:

$$Z_i \equiv R_i - R_f$$

¹. Mean-variance efficient portfolio
². Excess return

CAPM

$$E[Z_i] = \beta_{im} E[Z_m] \quad (1)$$

$$\beta_{im} = \frac{\text{Cov}[Z_i, Z_m]}{\text{Var}[Z_m]} \quad (2)$$

$$(1) \quad (2) \quad Z_m$$

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(E(Z_m))

i

: *i* . []

$$E[R_i] = E[R_{0m}] + \beta_{im} (E[R_m] - E[R_{0m}]) \quad (3)$$

m R_{0m} R_m

:

) . *m*

(

β_{im}

¹ Real returns

$$\beta_{im} = \frac{\text{Cov}[R_i, R_m]}{\text{Var}[R_m]} \quad ()$$

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: N $(N \times 1)$ Z_t

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t \quad ()$$

$$E(\varepsilon_t) = 0 \quad ()$$

$$E[\varepsilon_t \varepsilon_t'] = \Sigma \quad ()$$

$$E(Z_{mt}) = \mu_m \quad , \quad E[(Z_{mt} - \mu_m)^2] = \sigma_m^2 \quad ()$$

$$\text{Cov}[Z_{mt}, \varepsilon_t] = 0 \quad ()$$

Z_{mt} $(N \times 1)$ β

. . $(N \times 1)$ ε_t α t

α ()

() ()

Σ β α

:

$$\hat{\alpha} = \hat{\mu} - \hat{\beta} \hat{\mu}_m \quad ()$$

$$\hat{\beta} = \frac{\sum_{t=1}^T (Z_t - \hat{\mu})(Z_{mt} - \hat{\mu}_m)}{\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2} \quad ()$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T (Z_t - \hat{\alpha} - \hat{\beta}Z_{mt})(Z_t - \hat{\alpha} - \hat{\beta}Z_{mt})' \quad ()$$

$$(iid) \quad \begin{matrix} Z_{m1} \\ \vdots \\ Z_{mt} \dots Z_{m2} \end{matrix}$$

CAPM

CAPM

$$: \quad \gamma$$

$$E(R_t) = \gamma + \beta(E[R_{mt}] - \gamma) = (1-\beta)\gamma + \beta E[R_{mt}] \quad ()$$

$$(N \times 1) \quad \begin{matrix} R_t \\ \vdots \\ R_t \end{matrix} \quad \begin{matrix} \\ \\ N \end{matrix}$$

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t \quad ()$$

$$E[\varepsilon_t] = 0 \quad ()$$

$$E[\varepsilon_t \varepsilon_t'] = \Sigma \quad ()$$

$$E[R_{mt}] = \mu_m, \quad E[(R_{mt} - \mu_m)^2] = \sigma_m^2 \quad ()$$

$$\text{Cov}[R_{mt}, \varepsilon_t] = 0 \quad ()$$

¹. Identically and Independently distributed

$$\begin{matrix} R_{mt} & (N \times 1) & \beta \\ \cdot & \varepsilon_t & \alpha \\ (\) & (\) & t \end{matrix}$$

$$\alpha = (1-\beta)\gamma$$

$$\gamma \quad \beta$$

iid

$$\beta$$

$$(\)$$

$$\hat{\alpha} = \hat{\mu} - \hat{\beta}\hat{\mu}_m \quad (\)$$

$$\hat{\beta} = \frac{\sum_{t=1}^T (R_t - \hat{\mu})(R_{mt} - \hat{\mu}_m)}{\sum_{t=1}^T (R_{mt} - \hat{\mu}_m)^2} \quad (\)$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T (R_t - \hat{\alpha} - \hat{\beta}R_{mt})(R_t - \hat{\alpha} - \hat{\beta}R_{mt})' \quad (\)$$

[]

¹. Return interval

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \quad ()$$

$$S_\beta = \frac{1}{\sqrt{(T-1)}} \times \frac{S_\varepsilon}{S_m} \quad ()$$

¹ Standard error
² Standard deviation
³ Stationarity
⁴ Non-stationarity

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t

t

p

t

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¹. Premiums of factors

« » *t*

« »

t

t

t

t

t

t

t

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t

t

t

¹. Useless factor

t

t

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(OLS)

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OLS

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OLS

OLS

Gini

OLS

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(*DJIA*)

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¹. Risk aversion
². High-Frequency data
³. Dow Jones Industrial Average

$AR()$

$MA() \quad AR() \quad ARMA(\quad)$

$ARMA(\quad)$
 $ARMA(\quad)$
 $/ \quad /$

¹. Constant Beta
². Out of Sample

CAPM

CAPM

:

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t \quad ()$$

$$\begin{matrix} N & & (N \times 1) & & \beta & & N & & (N \times 1) & & R_t \\ (N \times 1) & & \varepsilon_t & & \alpha & & t & & & & R_{mt} \end{matrix}$$

(

() R_t

R_{mt}

α

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β

α

γ

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$$\alpha = (1 - \beta)\gamma$$

()

$R_{mt} \quad R_t$

CAPM

OLS

(ML) ()

OLS

$$z_t = \alpha + \beta z_{mt} + \varepsilon_t \quad (ML)$$

z_t

:

$$f(z_t | z_{mt}) = (2\pi)^{-\frac{T}{2}} |\Sigma|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}(z_t - \alpha - \beta z_{mt})' \Sigma^{-1} (z_t - \alpha - \beta z_{mt})\right] \quad ()$$

$t = 1, 2, 3, \dots, T$

T iid

:

$$f(z_1, z_2, \dots, z_T | z_{m1}, z_{m2}, \dots, z_{mT}) = \prod_{t=1}^T (2\pi)^{-\frac{T}{2}} |\Sigma|^{-\frac{1}{2}} \quad ()$$

$$\times \exp\left[-\frac{1}{2}(z_t - \alpha - \beta z_{mt})' \Sigma^{-1} (z_t - \alpha - \beta z_{mt})\right]$$

$\Sigma \quad \beta \quad \alpha$

$$L(\alpha, \beta, \Sigma) = -\frac{T}{2} \log(2\pi) - \frac{T}{2} \log |\Sigma| \quad ()$$

$$-\frac{1}{2} \sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})$$

$$\begin{matrix} () & Z_t \\ \Sigma & \beta & \alpha & L(\alpha, \beta, \Sigma) \end{matrix}$$

$$\frac{\partial L}{\partial \alpha} = \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt}) \right] \quad ()$$

$$\frac{\partial L}{\partial \beta} = \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt}) Z_{mt}' \right] \quad ()$$

$$\frac{\partial L}{\partial \Sigma} = -\frac{T}{2} \Sigma^{-1} + \frac{1}{2} \Sigma^{-1} \left[\sum_{t=1}^T (Z_t - \alpha - \beta Z_{mt})(Z_t - \alpha - \beta Z_{mt})' \right] \Sigma^{-1} \quad ()$$

$$() () ()$$

$$\hat{\alpha} = \hat{\mu} - \hat{\beta} \hat{\mu}_m \quad ()$$

$$\hat{\beta} = \frac{\sum_{t=1}^T (Z_t - \hat{\mu})(Z_{mt} - \hat{\mu}_m)'}{\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2} \quad ()$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T (Z_t - \hat{\alpha} - \hat{\beta} Z_{mt})(Z_t - \hat{\alpha} - \hat{\beta} Z_{mt})' \quad ()$$

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^T Z_t$$

$$\hat{\mu}_m = \frac{1}{T} \sum_{t=1}^T Z_{mt}$$

OLS

$$(T) \quad \frac{\sum e_i^2 / T}{\sum e_i^2 / (T-2)} \quad \sigma^2$$

σ^2

σ^2
CAPM

(GLS)

« »
OLS

GLS

(BLUE)

¹ Generalized Least Squares
² Best Linear Unbiased Estimator

(GMM)

(

v *t*- student Y_1, Y_2, \dots, Y_T
:

$$f(Y_t, v) = \frac{\Gamma[(v+1)/2]}{(\pi v)^{1/2} \Gamma(v/2)} \left[1 + (y_t^2/v)\right]^{-\frac{(v+1)}{2}} \quad ()$$

v

$$\hat{v} \equiv \arg \max_{\theta} \ln L_T(\theta) = \sum_{t=1}^T \ln f(y_t, v) \quad ()$$

r

$$\hat{\mu}_{r,T} = \frac{1}{T} \sum_{t=1}^T Y_t^r$$

$$\mu_r = \mu_r(v) \quad \mu_r$$

v

v

$a \times 1$

θ

$h \times 1$

w_t

θ_0

$h(\theta, w_t)$

r

θ

$$E[h(\theta_0, w_t)] = 0$$

$$y_t = (w'_1, w'_2, \dots, w'_T)$$

¹. Generalized method of moments

$$g(\theta, y_t) = \frac{1}{T} \sum_{t=1}^T h(\theta, w_t) \quad ()$$

$$\begin{array}{ccc}
 & g(\theta, y_t) & \theta \\
 \theta & & a > r \\
 & & a = r \\
 & \hat{\theta} & g(\theta, y_t) \\
 & & r > a
 \end{array}$$

$$Q_t(\theta, y_t) = [g(\theta, y_t)]' A_T [g(\theta, y_t)] \quad ()$$

$$A_T \quad r \times r \quad A_T$$

$$A_T = S^{-1} \quad ()$$

$$\begin{array}{ccc}
 & : & S \\
 & : & \\
 \hat{S}_T = \frac{1}{T} \sum_{t=1}^T [h(\theta_0, w_t)][h(\theta_0, w_t)]' & & () \\
 S^{-1} & & \theta_0
 \end{array}$$

$$\begin{array}{ccc}
 S_T^* & \hat{\theta} & \theta_0 \\
 & & \hat{S}_T & \theta_0 \\
 & & \hat{\theta}_{(j+1)} & \hat{\theta}_{(j)} \\
 & & : &
 \end{array}$$

$$\hat{S}_T = \hat{T}_{0T} + \sum_{v=1}^q \left(1 - \frac{v}{1+q} \right) (\hat{T}_{v,T} + \hat{T}'_{v,T}) \quad ()$$

$$T_{v,T} = \frac{1}{T} \sum_{t=v+1}^T [h(\hat{\theta}_T, w_t)][h(\hat{\theta}_T, w_{t-v})]'$$

$$a = K \quad \theta = \beta \quad \text{GMM} \quad \hat{S}_T \quad q$$

$$: \quad w_t = (Y_t, X_t')$$

$$X_t u_t = X_t (Y_t - X_t' \beta)$$

$$E(X_t u_t) = E[X_t (Y_t - X_t' \beta)] = 0$$

$$h(\theta, w_t) = X_t (Y_t - X_t' \beta)$$

$$g(\theta, y_t) = \frac{1}{T} \sum_{t=1}^T X_t (Y_t - X_t' \beta) = 0$$

$$\sum_{t=1}^T X_t Y_t = \sum_{t=1}^T X_t X_t' \beta$$

$$\hat{\beta}_{GMM} = \left[\sum_{t=1}^T X_t X_t' \right]^{-1} \left[\sum_{t=1}^T X_t Y_t \right] = (X'X)^{-1} X'Y \quad ()$$

GMM OLS

GMM

Z_t

$$Y_t = Z_t' \beta + u_t$$

$$u_t \quad X_t \quad Z_t$$

GMM $E(X_t u_t) = 0$

$$\theta = \phi \quad a = K \quad W_t = (Y_t, X_t', Z_t)$$

$$h(\theta, w_t) = X_t(Y_t - Z_t'\beta)$$

$$E[h(\theta_0, W_t)] = E[X_t(Y_t - Z_t'\beta_0)] = 0$$

$$g(\theta, y_t) = \frac{1}{T} \sum_{t=1}^T X_t(Y_t - Z_t'\beta) = 0$$

$$g(\theta, y_t) = 0$$

$$\sum_{t=1}^T X_t(Y_t - Z_t'\beta) = 0$$

$$\hat{\beta}_{GMM} = \left[\sum_{t=1}^T X_t Z_t' \right]^{-1} \left[\sum_{t=1}^T X_t Y_t \right] = (X'Z)^{-1} X'Y \quad ()$$

GMM

ML

GMM

(LAD)

(

()

$$\min_{\beta \in \mathbb{R}^k} \sum_{t=1}^T |Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}| \quad ()$$

:

$$\min_{\beta \in \mathbb{R}^k} \sum_{t=1}^T \rho_\tau(Y_t - \xi(x_t, \beta))$$

¹. Least Absolute Deviation
². Quantile regression

$$\xi_t = Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}$$

$$\rho_\tau(\cdot) \quad \xi = f(x_{1t}, x_{2t}, \dots, x_{kt}; \beta_1, \beta_2, \dots, \beta_k)$$

$$\rho_\tau(u) = u(\tau - 1(u < 0))$$

$$Q_\tau = \min_{\xi \in \mathbb{R}} \left\{ \sum_{t: Y_t \geq \xi} \tau |Y_t - \xi| + \sum_{t: Y_t < \xi} (1 - \tau) |Y_t - \xi| \right\} \quad ()$$

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(NP) (

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T

T(T+1)/2

:

$$\min \sum_{t=1}^T [\text{rank}(Y_t - \beta_1 - \beta_2 X_{2t} - \dots - \beta_k X_{kt}) - T(T+1)/2] p_t \quad ()$$

[]

$$(\text{rank}(e) - (T+1)/2)' e \quad ()$$

:

$$e = y - x\hat{\beta}$$

$$x = [x_2, x_3, \dots, x_k]; \hat{\beta}' = [\hat{\beta}_2, \dots, \hat{\beta}_k]$$

e

¹. Histogram
². Kernel
³. Rank Regression
⁴. Nadaraia- Watson

$$f(\hat{\beta}) = (\text{rank}(y - x\hat{\beta}) - (T+1)/2)'(y - x\hat{\beta}) \quad (49)$$

β

$$\hat{\beta}^i = \hat{\beta}^0 + pd$$

$$\delta \quad \hat{\beta}^i - \hat{\beta}^{i-1} < \delta$$

: d p 1e-10 1e-6

$$z = y - x\hat{\beta}^0; u^0 = \text{rank}(z) - 0.5(T+1); x_c = x - \bar{x}$$

$$d = (x_c' x_c)^{-1} x_c' u^0; w = xd$$

p

w

$$|w_i - w_j| / \sum |w_i - w_j| \quad (z_i - z_j) / (w_i - w_j)$$

LAD NP, GLS, ML, GMM

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LAD NP, GLS, GLS GMM

LAD NP, GLS, GLS GMM

()

()

()

ML

GLS

GLS

()

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¹. Low Risk
². High Risk

:()

		/	
/		GLS	
/		ML	
/		GMM	
/		NP	
/ /		LAD	
/		Black	
/		Sharp	
/		GLS	
/		ML	
/		GMM	
/		NP	
/		LAD	
/		Black	
/		Sharp	

H_0

$(\sum e_i^2)$

:()

		/	
/		GLS	black
/		ML	
/		GMM	
/		NP	
/		LAD	
/			
/			
/		GLS	sharp
/		ML	
/		GMM	
/		NP	
/		LAD	
/			
/			

$$\sum e_i^2 \quad NP$$

$$\sum e_i^2 \quad ML$$

NP

LAD

ML

GLS NP

ML

GMM LAD

:()

			/
/		Black	GLS
/		Sharp	
/			
/			GMM
/		Black	
/		Sharp	
/			
/			ML
/		Black	
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/			
/			LAD
/		Black	
/		Sharp	
/			
/			NP
/		Black	
/		Sharp	
/			

R

CAPM
CAPM

$\sum e_i^2$ R
CAPM

LAD NP ML GMM GLS

OLS

NP

ML

LAD

R

GMM

GLS

GMM

NP

$\sum e_i^2$

NP

$\sum e_i^2$

1. Barrodale, I. and Roberts, F. D. K. "Solution of an Overdetermined System of Equations in the l_1 Norm," Communications of the ACM, 17(6), 319- 320, (1974).
2. Birkes, D. and Dodge, Y. "Alternative Methods of Regression", Wiley. (1993).
3. Black, F.; Jensen, M. C. and sholes, M. "the Capital Asset Pricing Model: Some Empirical Tests", in M.C. Jensen. Studies in Theory of capital Markets, ed. New york: praeger, (1972).
4. Campbell, John Y.; Andrew, W. Lo and A. Craig Mackinlay. (1997). "The Econometrics of Financial Markets", Princeton university press, princeton, Newjersey.
5. Chi- Fu, Huang and Robert H., Litzenberger. (1988). "Foundations for Financial Economics", North- Holland.
6. Cuthbertson, Keith. and Nitzsche, Dirk. (2005). "Quantitative Financial Economics", 2nd ed., John Wiley & Sons.
7. Eisenbei, Maik, Kauermann, Goranand and willi Semmler. (2003). "Estimating Beta- coefficients of German Stock Data: A Non-Parametric Approach", universitat Munster.
8. Jagannathan, Ravi and Zhenyu Wang. (1998). "An Asymptotic thory for Estimaling Beta- Pricing Models Using Cross-Sectional Regaression", journal of Finance, 4, 1285- 1309.
9. Kan, Raymond. and C. Zhang. (1997). "Two-Pass Tests of Asset Pricing

Models with Useless Factors.”, journal of Finance, forthcoming.

10. Koenker, Roger W. and Vasco D'Orey. (1987). “Algorithm AS 229: Computing Regression Quantiles”, Applied Statistics, 36(3), 383- 393.
11. Lintner, John. (1965). “The valuation of Risk Assets and the selection of Risky Investments in stock portfolios an capital Budgets”, Review of Economics and Statistics, 47, 13- 37.
12. Markowitz, H. M. (1959). “Portfolio Selection”, journal of finance
13. Philip, R. Daves, Michael, C. Ehrhardt and Robert A.Kunkel, (2000). “Estimating Systematic Risk: The Choice of Return interval and Estimation Period”, journal of Financial and Strategic Decisions, 13 (1), 7- 13.
14. Pagan, A. and Ullah, A. (1999). “Nonparametric Econometrics.” Cambridge University Press.
15. Qianqiu, Liu. (2002). “Estimating Betas From High- Frequency Data”, working paper, Department of Financial Economics and institutions, college of Business Administration, university of Hawaii.
16. Shalit, Haim and Shlomo, Yitzhaki. (2001). “Estimating Beta”, Department of economics, Ben-Gurion university of the Negev, Beer sheva 84105 Israel.
17. Sharp, William F. (1964). “Capital Asset Prices: a Theory of Market Equilibrium under Conditions of Risk”, journal of finance, 19, 425- 442,
18. Terence, C. Mills. (1999). “The Econometric Modelling of Financial Time Series”, 2nd ed., Cambridge university press, Cambridge.